

## 第一部分：選擇題

### 一、單一選擇題

#### 答案

1.(D) 2.(A) 3.(D) 4.(B)

### 二、多重選擇題

5.(A)(C)(E) 6.(A)(B)(D)(E) 7.(C)(D) 8.(B)(C)(E) 9.(A)(B)(C) 10.(A)(B)(D)

## 第二部分：填充題

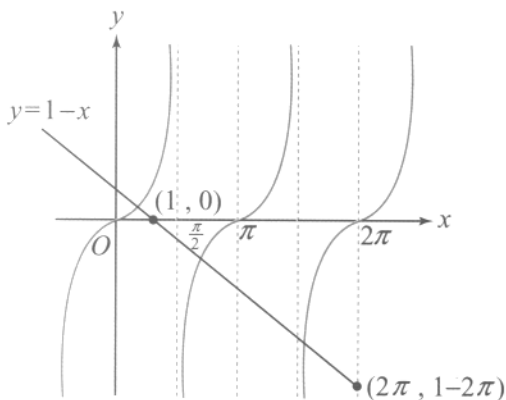
11. 150      12. 9      13. 20      14. 24      15. 24  
 16.  $\frac{1-\sqrt{5}}{2}$       17.  $50\sqrt{7}$       18.  $2x+5$       19.  $\frac{(x-2)^2}{9} + \frac{(y-0)^2}{5} = 1$       20. 109

#### 解析

## 第一部分：選擇題

### 一、單一選擇題

1. 直線  $y=1-x$  過  $(1, 0)$ ,  $(2\pi, 1-2\pi)$  兩點



由上圖知兩圖形有 3 個交點，故選(D)。

2. 設  $\alpha$  為  $x^2+ax+3-i=0$  之另一根，

$$\begin{cases} (1-i)+\alpha=-a \\ (1-i)\cdot\alpha=3-i \end{cases} \Rightarrow a=2+i,$$

$$\therefore -a=1-i+2+i=3, a=-3,$$

故選(A)。

3.  $P(A)=\frac{1}{2}$ ,  $P(B)=\frac{1}{3}$ , 設  $P(A\cup B)=p$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$\therefore P(A\cap B)\geq 0$$

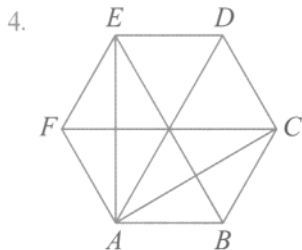
$$\therefore p\leq \frac{1}{2}+\frac{1}{3}=\frac{5}{6}\cdots\cdots\textcircled{1}$$

$$\text{又} \begin{cases} P(A\cup B)\geq P(A) \\ P(A\cup B)\geq P(B) \end{cases}$$

$$\Rightarrow \begin{cases} p \geq \frac{1}{2} \\ p \geq \frac{1}{3} \end{cases} \Rightarrow p \geq \frac{1}{2} \cdots \cdots \textcircled{2}$$

由①, ②知,  $\frac{1}{2} \leq p \leq \frac{5}{6}$ ,

故選(D).



$$(1) \overrightarrow{AB} \cdot \overrightarrow{AB} = |\overrightarrow{AB}|^2.$$

$$(2) \overrightarrow{AB} \cdot \overrightarrow{AC} = |\overrightarrow{AB}| |\overrightarrow{AC}| \cos 30^\circ \\ = \sqrt{3} |\overrightarrow{AB}|^2 \times \frac{\sqrt{3}}{2} = \frac{3}{2} |\overrightarrow{AB}|^2.$$

$$(3) \overrightarrow{AB} \cdot \overrightarrow{AD} = |\overrightarrow{AB}| \cdot |\overrightarrow{AD}| \cos 60^\circ \\ = |\overrightarrow{AB}|^2.$$

$$(4) \overrightarrow{AB} \cdot \overrightarrow{AE} = 0.$$

$$(5) \overrightarrow{AB} \cdot \overrightarrow{AF} = |\overrightarrow{AB}| |\overrightarrow{AF}| \cos 120^\circ < 0,$$

$\therefore \overrightarrow{AB} \cdot \overrightarrow{AC}$  之值最大, 故選(B).

## 二、多重選擇題

$$5. (2k)^2 = 4k^2$$

$$(2k+1)^2 = 4k^2 + 4k + 1 = 4k(k+1) + 1$$

觀察餘數

$$513 = 4 \times 128 + 1 \cdots \cdots \textcircled{1}$$

$$226 = 4 \times 56 + 2 \cdots \cdots \textcircled{2}$$

$$216 = 4 \times 54 + 0 \cdots \cdots \textcircled{3}$$

$$154 = 4 \times 38 + 2 \cdots \cdots \textcircled{4}$$

$$145 = 4 \times 36 + 1 \cdots \cdots \textcircled{5}$$

$$\textcircled{2} \text{ 與 } \textcircled{4} \Rightarrow 4(56 + 38 + 1) = 4 \times \underline{95}$$

非完全平方數, 故不可能  $\xrightarrow{\quad}$

①與③

$$\Rightarrow 4(128 + 54) + 1 = 4 \times \underline{13 \times 14} + 1 \\ (\text{成立}) \quad k(k+1)$$

③與⑤

$$\Rightarrow 4(54 + 36) + 1 = 4 \times \underline{9 \times 10} + 1 \\ (\text{成立}) \quad k(k+1)$$

故選(A)(C)(E).

$$6. L_1: ax - 4y = 1, \quad m_1 = \frac{a}{4},$$

$$L_2: (a+1)x + 3y = 2, \quad m_2 = \frac{-a-1}{3},$$

$$L_3: x - 2y = 3, \quad m_3 = \frac{1}{2}.$$

$$(1) L_1 \perp L_2, \quad m_1 \cdot m_2 = -1$$

$$\Rightarrow a = 3, -4.$$

$$(2) L_2 \perp L_3, \quad m_2 \cdot m_3 = -1 \Rightarrow a = 5.$$

$$(3) L_1 \perp L_3, \quad m_1 \cdot m_3 = -1 \Rightarrow a = -8.$$

故選(A)(B)(D)(E).

$$7. (1) \sin 50^\circ = \cos 40^\circ > \cos 50^\circ.$$

$$(2) \tan 50^\circ = \cot 40^\circ > \cot 50^\circ.$$

$$(3) \sec^2 50^\circ = \tan^2 50^\circ + 1$$

$$\therefore \sec^2 50^\circ > \tan^2 50^\circ,$$

$$(\because \sec 50^\circ > 0, \tan 50^\circ > 0)$$

$$\therefore \sec 50^\circ > \tan 50^\circ.$$

$$(4) \sin 230^\circ = -\sin 50^\circ,$$

$$\cos 230^\circ = -\cos 50^\circ.$$

$$\therefore \sin 50^\circ > \cos 50^\circ$$

$$\Rightarrow -\sin 50^\circ < -\cos 50^\circ,$$

$$\therefore \sin 230^\circ < \cos 230^\circ.$$

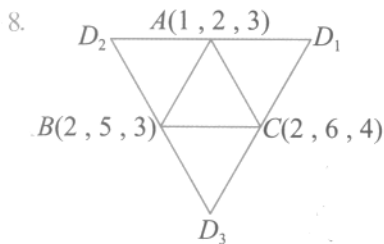
$$(5) \tan 230^\circ = \tan 50^\circ,$$

$$\cot 230^\circ = \cot 50^\circ,$$

$$\therefore \tan 50^\circ > \cot 50^\circ,$$

$$\therefore \tan 230^\circ > \cot 230^\circ.$$

故選(C)(D).



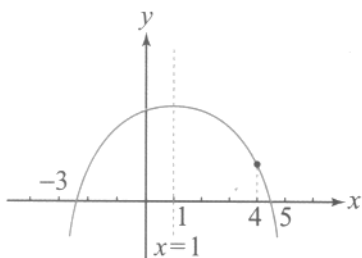
$$D_1(1, 3, 4),$$

$$D_2(1, 1, 2),$$

$$D_3(3, 9, 4),$$

故選(B)(C)(E).

9.  $f(4) > 0$ ,  $f(5) < 0$ , 且軸為  $x = 1$ ,



故圖形如上.

$$\therefore f(0) > 0, f(-1) > 0, f(-2) > 0,$$

$$f(-3) < 0, f(-4) < 0,$$

故選(A)(B)(C).

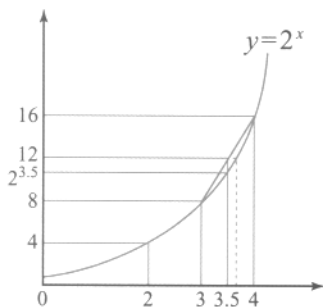
10.

$x$	1	2	3	4
$f(x)$	2	4	8	16

(1)  $f(x) = 2^x$ .

(2)  $f(5) = 2^5 = 32 > 30$ .

(3) 如下圖,  $2^{3.5} < 12$  故時間一定會超過 1.5 個月.



(4)  $2^a = 2$ ,  $2^b = 3$ ,  $2^c = 6$ .

$$2^{a+b} = 2^c, \therefore t_1 + t_2 = t_3.$$

(5) 第 1 到第 3 個月的蔓延平均速度

$$= \frac{8-2}{3-1} = 3,$$

第 2 到第 4 個月的蔓延平均速度

$$= \frac{16-4}{4-2} = 6.$$

故選(A)(B)(D).

## 第二部分：填充題

11. 百十個

$$a \ b \ c$$

$$(a, c) = (2, 0), (3, 1), \dots, (9, 7)$$

$\Rightarrow$  8 個

$$(8 \times 2 - 1) \times 10 = 150$$

$\uparrow$   $\uparrow$   $\uparrow$   
 $a, c$  可對調  
 $b$  有 10 種情形  
 扣掉  $(0, 2)$

12.  $\frac{a}{2^1} + \frac{b}{2^2} + \frac{a}{2^3} + \frac{b}{2^4} + \dots + \frac{a}{2^{2n-1}} + \frac{b}{2^{2n}} + \dots$

$$= \left( \frac{a}{2^1} + \frac{a}{2^3} + \frac{a}{2^5} + \dots \right) + \left( \frac{b}{2^2} + \frac{b}{2^4} + \dots \right)$$

$$= \frac{\frac{a}{2}}{1 - \frac{1}{4}} + \frac{\frac{b}{4}}{1 - \frac{1}{4}} = \frac{2}{3}a + \frac{b}{3} = 3$$

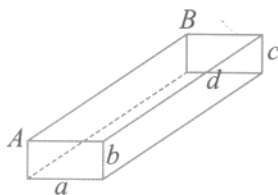
$$\Rightarrow 2a + b = 9.$$

13. 若甲、乙、丙單獨生產各需  $x$ 、 $y$ 、 $z$  小時,

$$\begin{cases} 10\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 1 \\ 15\left(\frac{1}{y} + \frac{1}{z}\right) = 1 \\ \frac{15}{x} + \frac{30}{z} = 1 \end{cases}$$

$$\begin{cases} \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{10} \\ \frac{1}{y} + \frac{1}{z} = \frac{1}{15} \\ \frac{1}{x} + \frac{2}{z} = \frac{1}{15} \end{cases} \Rightarrow y = 20.$$

14.



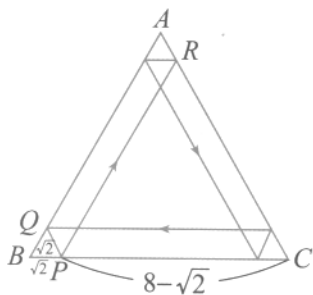
與  $\overline{AB}$  歪斜的稜線有  $a, b, c, d$  四個，

$$\therefore 12 \times 4 \times \frac{1}{2} = 24$$

↑ 重複算了兩次

( $\overline{AB}, a$ ) 與 ( $a, \overline{AB}$ ) 是相同的。

15.



$$\overline{PR} = \overline{PC} = 8 - \sqrt{2},$$

$$\overline{PQ} = \overline{BP} = \sqrt{2},$$

$\therefore$  路徑長

$$= 3 \times (8 - \sqrt{2}) + (3 \times \sqrt{2}) = 24.$$

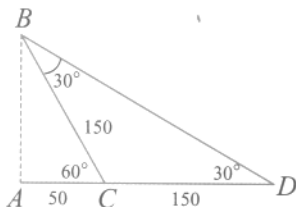
$$16. a_4 = a_3 + a_2$$

$$= (a_2 + a_1) + a_2$$

$$= 2a_1r + a_1, \quad a_1 = 1 \text{ 代入,}$$

$$2 - \sqrt{5} = 2r + 1 \Rightarrow r = \frac{1 - \sqrt{5}}{2}.$$

17.



$$\angle CBD = 30^\circ,$$

$$\therefore \overline{CB} = \overline{CD} = 150.$$

$\triangle ABC$  中，由餘弦定理知，

$$\overline{AB}^2$$

$$= 50^2 + 150^2 - 2 \times 50 \times 150 \cos 60^\circ,$$

$$\overline{AB} = 50\sqrt{1+9-3} = 50\sqrt{7}.$$

$$18. \text{設 } f(x) = (x^2 + x + 1)Q(x) + (ax + b),$$

$$(x+1)f(x)$$

$$= (x^2 + x + 1)[(x+1)Q(x)]$$

$$+ (x+1)(ax + b)$$

$$= (x^2 + x + 1)[(x+1)Q(x)]$$

$$+ [ax^2 + (a+b)x + b]$$

$$= (x^2 + x + 1)[(x+1)Q(x)]$$

$$+ a(x^2 + x + 1) + bx + (b-a)$$

$$= (x^2 + x + 1)[(x+1)Q(x) + a]$$

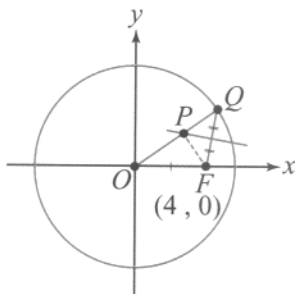
$$+ bx + (b-a),$$

$$\therefore bx + (b-a) = 5x + 3,$$

$$\begin{cases} b = 5 \\ b - a = 3 \end{cases} \Rightarrow \begin{cases} a = 2 \\ b = 5 \end{cases}$$

$\therefore$  餘式為  $2x + 5$ .

19.



連  $\overline{PF}$ ,  $\overline{PQ} = \overline{PF}$ .

$$\therefore \overline{OP} + \overline{PQ} = \overline{OP} + \overline{PF} = 6,$$

由橢圓的定義知：

$O, F$  為兩焦點，中心  $(2, 0)$ ,

$$c = 2, \quad 2a = 6, \quad a = 3,$$

$$b^2 = a^2 - c^2 = 5,$$

$$\text{故方程式 } \frac{(x-2)^2}{9} + \frac{(y-0)^2}{5} = 1.$$

## 20. 拉氏指數

$$\begin{aligned} &= \frac{\Sigma(\text{計算期價格} \times \text{基期消費量})}{\Sigma(\text{基期價格} \times \text{基期消費量})} \times 100 \\ &= \frac{\Sigma(76 \text{ 年的價格} \times 70 \text{ 年的消費量})}{\Sigma(70 \text{ 年的價格} \times 70 \text{ 年的消費量})} \times 100 \\ &= \frac{(16 \times 45) + (97 \times 5) + (74 \times 0.5) + (15 \times 4) + (13 \times 3) + (54 \times 0.8)}{(7.6 \times 45) + (49 \times 5) + (36 \times 0.5) + (5.6 \times 4) + (4.7 \times 3) + (25 \times 0.8)} \times 100 \\ &= \frac{1384.2}{661.5} \times 100 = 209. \end{aligned}$$

$$I_{\frac{76}{70}} = 209, \quad I_{\frac{70}{70}} = 100,$$

76 年的費用比 70 年的費用高出的百分比

$$\begin{aligned} &= \frac{I_{\frac{76}{70}} - I_{\frac{70}{70}}}{I_{\frac{70}{70}}} \times 100\% = \frac{209 - 100}{100} \times 100\% \\ &= 109\%. \end{aligned}$$