答案與解析 Answer

答案

一、單選題

1.(3)　2.(1)　3.(4)　4.(2)　5.(5)

二、多選題

6.(1)(2)　7.(2)(3)(5)　8.(2)　9.(1)(5)　10.(2)(5)　11.(2)(3)(4)(5)

三、填充題

⑫ 3　⑬ 8　⑭ 2　⑮ 5　⑯ 2　⑰ 3　⑱ 6　⑲ 3　⑳ 2　㉑ −　㉒ 1　㉓ 2　㉔ 5

㉕ 1　㉖ 6　㉗ 3　㉘ 4　㉙ 3　㉚ 2　㉛ 1　㉜ 5　㉝ 5　㉞ 6

解析

一、單選題

1. $43659 = 3^4 \times 7^2 \times 11$,

   ∴有 3 個質因數.

2. 原式 $= (\frac{20 \times 21}{2})^2 - (\frac{10 \times 11}{2})^2$

   $= 210^2 - 55^2 = 41075$.

3. $\frac{r}{R} = \frac{\frac{1}{C_5^{39}}}{\frac{1}{C_6^{42}}}$

   $= \frac{42 \times 41 \times 40 \times 39 \times 38 \times 37}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$

   $\times \frac{5 \times 4 \times 3 \times 2 \times 1}{39 \times 38 \times 37 \times 36 \times 35} \doteqdot 9$.

4. $\log_7 a = 11 \Rightarrow a = 7^{11}$,

   $\log_7 b = 13 \Rightarrow b = 7^{13}$.

   設 $\log_7(a+b) = x \Rightarrow a + b = 7^x$,

   $7^{11} + 7^{13} = 7^x$, $(1 + 7^2) \times 7^{11} = 7^x$,

   $50 \times 7^{11} = 7^x$,

   ∴ 7^x 最接近 $7^2 \times 7^{11} = 7^{13}$,

   ∴ $\log_7(a+b)$ 最接近 13.

5. 設原始分數 y_i，調整後的分數 x_i，即

   $x_i = 10\sqrt{y_i}$,

   $15 = \sqrt{\frac{1}{99}(\Sigma x_i^2 - 100 \times 65^2)}$,

   $\Sigma x_i^2 = 99 \times 15^2 + 100 \times 65^2$,

   $\Sigma(10\sqrt{y_i})^2 = 99 \times 15^2 + 100 \times 65^2$,

   $\Sigma y_i \doteqdot 15^2 + 65^2$,

   $M = \frac{1}{100}\Sigma y_i \doteqdot \frac{4450}{100} = 44.5$,

   即 $44 \le M < 45$.

二、多選題

6. 由向量加減的圖示法判斷(1), (3), (5)，又(2), (4)可由 $\alpha\overrightarrow{OA} + \beta\overrightarrow{OB}$,

   $\begin{cases} 若\alpha + \beta = 1，終點落在 \overline{AB} 上, \\ 若\alpha + \beta > 1，終點落在陰影區域, \\ 若\alpha + \beta < 1，終點落在\triangle OAB 內. \end{cases}$

7. m_{CD} 最大, m_{BC} 最小, $m_{BC} = -m_{CD}$,

$\overline{AB}$ 與 $\overline{BC}$ 不垂直,

$\therefore m_{AB} \times m_{BC} \neq -1$,

又 $m_{AB} = -m_{AD}$ 且 $m_{CD} > m_{AD}$,

$\therefore m_{CD} + m_{DA} = m_{CD} - m_{AB} > 0$.

8. $A(-1, 2, 0)$, $B(3, 0, 2)$,

$$\overleftrightarrow{AB}: \begin{cases} x = -1 + 4t \\ y = 2 - 2t \\ z = 0 + 2t \end{cases}, \quad t \in \boldsymbol{R},$$

其中只有$(1, 1, 1)$在 $\overleftrightarrow{AB}$ 上, 其餘四點皆不在 $\overleftrightarrow{AB}$ 上.

9. (1) $0 < \theta < \dfrac{\pi}{4} \Rightarrow \sin\theta < \cos\theta$.

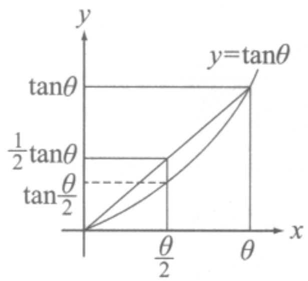
(2) $\cos\theta < 1 \Rightarrow \dfrac{\sin\theta}{\cos\theta} > \dfrac{\sin\theta}{1}$

$\Rightarrow \tan\theta > \sin\theta$.

(3)無法判斷.

(4) $0 < 2\theta < \dfrac{\pi}{2}$, $\sin 2\theta$ 與 $\cos 2\theta$ 無法判斷.

(5)



$\therefore \tan\dfrac{\theta}{2} < \dfrac{1}{2}\tan\theta$.

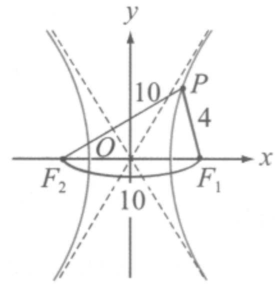
10. Γ: $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$,

$a^2 = 9$, $b^2 = 16$, $c^2 = 25$,

$a = 3$, $c = 5$, $|\overline{PF_2} - \overline{PF_1}| = 6$,

$\overline{F_1F_2} = 2c = 10$,

(1)

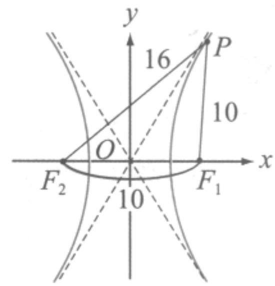


$\overline{PF_2} = \overline{F_1F_2} = 10$,

又 $\overline{PF_1} = \overline{PF_2} - 6 = 4$,

周長 $= 10 + 10 + 4 = 24$.

(2)



$\overline{PF_1} = \overline{F_1F_2} = 10$,

$\overline{PF_2} = 16$,

周長 $= 10 + 10 + 16 = 36$.

11. 觀察截平面圖形,

$\overline{PA} + \overline{PB} = 14$……橢圓定義,

$a = 7$, $\overline{AB} = 2c = 10$, $c = 5$,

$\overline{AE} = 5\sqrt{2}$, $\overline{AD} = 7$,

$\overline{AE} > \overline{AD}$, $\therefore D$ 在圓內,

圖形如下.

(1) P 不可能在 $\overline{AB}$ 上.

(2) P 可能在 $\overleftrightarrow{AB}$ 上, 如點 P_1.

(3) P 可能在球面上，如點 P_3 .

(4) P 可能在球內部，如點 P_4 .

(5) P 可能在球外部，如點 P_2 .

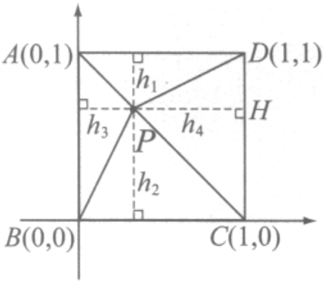
三、填充題

A.

1	$+1$	$+1$	$+p$	$+2$	$+q$	
	-1	$+0$	$+1$	$-p-1$		-1
		-2	$+0$	$+2$	$-2p-2$	-2
1	$+0$	-1	$+(p+1)$	$+(-p+3)$	$+(q-2p-2)$	

$\therefore \begin{cases} -p+3=0 \\ q-2p-2=0 \end{cases} \Rightarrow \begin{cases} p=3 \\ q=8 \end{cases}.$

B.



$\triangle PDA : \triangle PBC = 1:2$

$\Rightarrow \quad h_1 : h_2 = 1:2,$

$\triangle PAB : \triangle PCD = 2:3$

$\Rightarrow \quad h_3 : h_4 = 2:3,$

又 $h_1 + h_2 = 1, \quad \therefore h_2 = \dfrac{2}{3},$

$h_3 + h_4 = 1, \quad \therefore h_3 = \dfrac{2}{5},$

$\therefore P\left(\dfrac{2}{5}, \dfrac{2}{3}\right).$

C. 最後落在 $+4$ 處，表示共有 5 正 1 負，

$\dfrac{6!}{5!\,1!} = 6$ 種不同跳法.

D. $z = 1-i, \quad z^2 = -2i,$

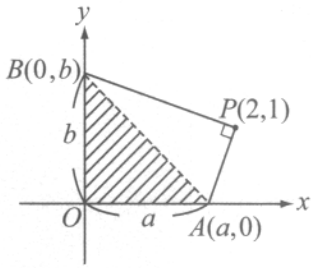
$z^{10} = (-2i)^5 = -32i,$

$1 + z + z^2 + \cdots + z^9$

$= \dfrac{1 \times (1-z^{10})}{1-z} = \dfrac{1-(-32i)}{1-(1-i)}$

$= \dfrac{1+32i}{i} = 32 - i.$

E. 設 $A(a, 0), \quad B(0, b),$



$\overrightarrow{PA} = (a-2, -1),$

$\overrightarrow{PB} = (-2, b-1),$

$\overrightarrow{PA} \perp \overrightarrow{PB} \quad \Rightarrow \quad \overrightarrow{PA} \cdot \overrightarrow{PB} = 0,$

$-2a + 4 - b + 1 = 0,$

$\therefore 2a + b = 5,$

$\dfrac{2a+b}{2} \ge \sqrt{2ab}, \quad \dfrac{5}{2} \ge \sqrt{2ab},$

$\dfrac{25}{4} \ge 2ab, \quad ab \le \dfrac{25}{8},$

$\triangle OAB = \dfrac{1}{2} \times a \times b$

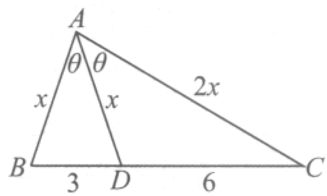
$\le \dfrac{1}{2} \times \dfrac{25}{8}$

$= \dfrac{25}{16}$ —— 最大值.

F. $\overline{AB} : \overline{AC} = \overline{BD} : \overline{CD} = 1:2,$

設 $\overline{AB} = x,$

$\therefore \overline{AD} = x, \quad \overline{AC} = 2x,$



$\triangle ABD$ 中， $\cos\theta = \dfrac{x^2 + x^2 - 9}{2 \times x \times x},$

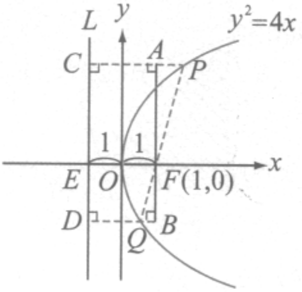
$\triangle ACD$ 中， $\cos\theta = \dfrac{x^2 + 4x^2 - 36}{2 \times x \times 2x},$

$\therefore \dfrac{2x^2 - 9}{2x^2} = \dfrac{5x^2 - 36}{4x^2},$

$\therefore x^2 = 18,$

$\therefore \cos\angle BAD = \cos\theta = \dfrac{27}{36} = \dfrac{3}{4}.$

G. 如下圖,



由拋物線定義知: $\overline{PF}=\overline{PC}$, $\overline{QF}=\overline{QD}$,

$2\overline{PF}=3\overline{QF}$, $\overline{PF}:\overline{QF}=3:2$.

設 $\overline{PF}=\overline{PC}=3t$, $\overline{QF}=\overline{QD}=2t$,

$\overline{AC}=\overline{FE}=\overline{BD}=2$,

$\therefore \overline{AP}=3t-2$, $\overline{BQ}=2-2t$,

$\triangle APF \sim \triangle BQF$,

$\therefore \dfrac{\overline{AP}}{\overline{BQ}}=\dfrac{\overline{PF}}{\overline{QF}}=\dfrac{3}{2}$, $\dfrac{3t-2}{2-2t}=\dfrac{3}{2}$,

$t=\dfrac{5}{6}$.

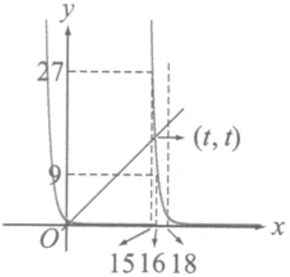
$\overline{PC}=3t=\dfrac{5}{2}$,

$\therefore P$ 的 x 坐標為 $\dfrac{5}{2}-1=\dfrac{3}{2}$.

H. **〈方法一〉**利用圖形

方程式 $x\cdot 3^x=3^{18}$ 的根,

就是 $\begin{cases} y=x \\ y=3^{18-x} \end{cases}$ 兩圖形交點,



x	3^{18-x}
14	81
15	27
16	9
17	3

由圖知 $t=15.\cdots$, $k\le t<k+1$,

$\therefore k=15$.

〈方法二〉勘根定理

$x\cdot 3^x=3^{18} \Rightarrow x=3^{18-x}$

$\Rightarrow \dfrac{3^{18-x}}{x}=1$,

$\therefore \dfrac{3^{18-x}}{x}-1=0$,

令 $f(x)=\dfrac{3^{18-x}}{x}-1$,

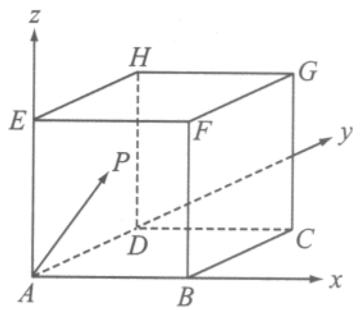
x	$f(x)$	
14	$\dfrac{81}{14}-1$	+
15	$\dfrac{27}{15}-1$	+ ←
16	$\dfrac{9}{16}-1$	−
17	$\dfrac{3}{17}-1$	−

方程式 $f(x)=0$ 在 15~16 之間有一根,

$\therefore k=15$.

I. 將正立方體平移至原點,

如下圖,



$\overrightarrow{AB}=(1,0,0)$, $\overrightarrow{AD}=(0,1,0)$,

$\overrightarrow{AE}=(0,0,1)$,

$\overrightarrow{AP}=\dfrac{3}{4}\overrightarrow{AB}+\dfrac{1}{2}\overrightarrow{AD}+\dfrac{2}{3}\overrightarrow{AE}$

$=(\dfrac{3}{4},\dfrac{1}{2},\dfrac{2}{3})$,

$\therefore P(\dfrac{3}{4},\dfrac{1}{2},\dfrac{2}{3})$,

$d(P,\overleftrightarrow{AB})=d(P, x\text{ 軸})$

$=\sqrt{\dfrac{1}{4}+\dfrac{4}{9}}=\dfrac{5}{6}$.